Dronacharya College of Engineering

INTELLIGENT SYSTEMS (CSE-303-F)

Section A

UNINFORMED AND INFORMED SEARCH

LECTURE 1

Outline

- Motivation
- Technical Solution
  - Uninformed Search
    - Depth-First Search
    - Breadth-First Search
  - Informed Search
    - Best-First Search
    - Hill Climbing
    - A*
- Illustration by a Larger Example
- Extensions
- Summary

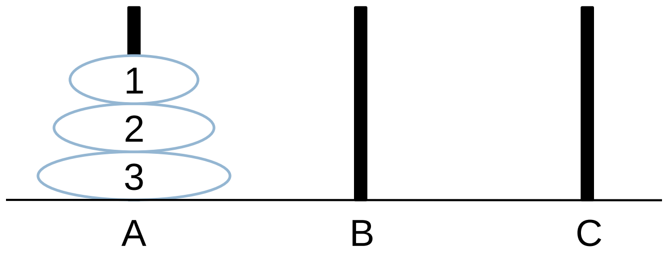
Motivation

- One of the major goals of AI is to help humans in solving complex tasks
  - How can I fill my container with pallets?
  - Which is the *shortest* way from Milan to Innsbruck?
  - Which is the *fastest* way from Milan to Innsbruck?
  - How can I *optimize* the load of my freight to *maximize* my revenue?
  - How can I solve my Sudoku game?
  - What is the sequence of actions I should apply to win a game?
- Sometimes finding a solution is not enough, you want the optimal solution according to some “cost” criteria
- All the example presented above involve looking for a plan
- A plan that can be defined as the set of operations to be performed of an initial state, to reach a final state that is considered the goal state
- Thus we need efficient techniques to *search* for paths, or sequences of actions, that can enable us to reach the goal state, i.e. to find a plan
- Such techniques are commonly called *Search Methods*

Examples of Problems: Towers of Hanoi

- 3 pegs A, B, C
- 3 discs represented as natural numbers (1, 2, 3) which correspond to the size of the discs
- The three discs can be arbitrarily distributed over the three pegs, such that the following constraint holds:

  d_i is on top of $d_j \rightarrow d_i < d_j$
- Initial status: ((123)()())
- Goal status: (()()(123))



Operators:

Move *disk* to *peg*

Applying: Move 1 to C (1 → C)

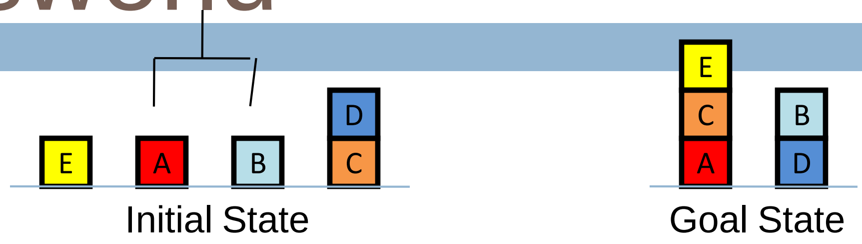
to the initial state ((123)()())

a new state is reached

((23)()(1))

Cycles may appear in the solution!

Examples of Problems: Blocksworld



- Objects: blocks
- Attributes (1-ary relations): cleartop(x), ontable(x)
- Relations: on(x,y)
- Operators: puttable(x) where x must be cleartop; put(x,y), where x and y must be cleartop

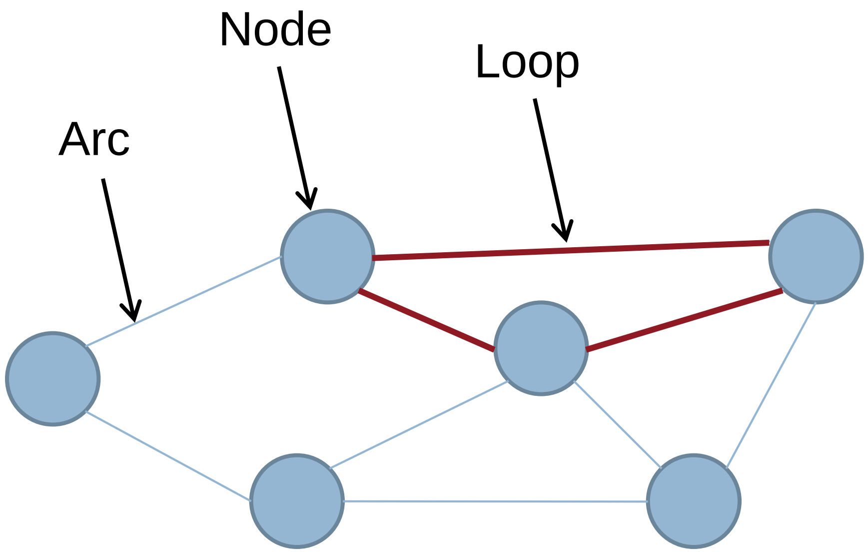
- Initial state:
  - ontable(E), cleartop(E)
  - ontable(A), cleartop(A)
  - ontable(B), cleartop(B)
  - ontable(C)
  - on(D,C), cleartop (D)
- Applying the move put(E,A):
  - on(E,A), cleartop(E)
  - ontable(A)
  - ontable(B), cleartop(B)
  - ontable(C)
  - on(D,C), cleartop (D)

TECHNICAL SOLUTION

Search Methods

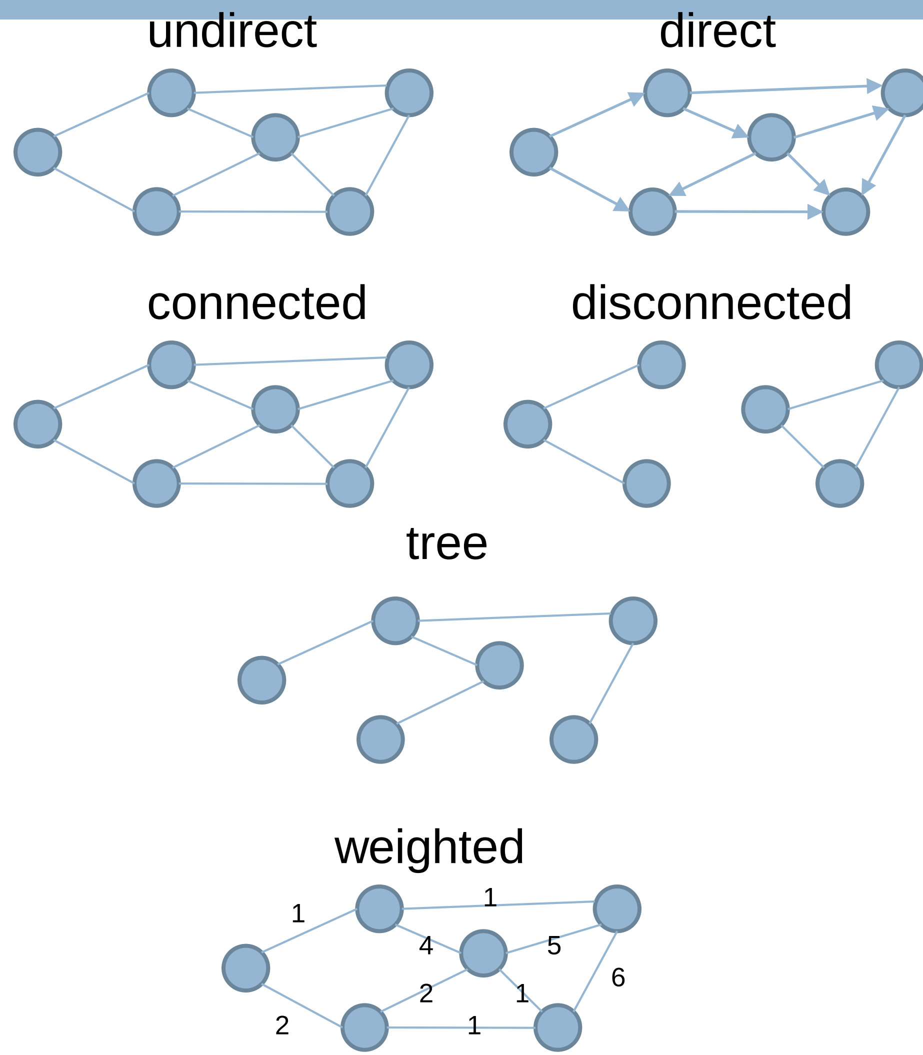
Search Space Representation

- Representing the search space is the first step to enable the problem resolution
- Search space is mostly represented through graphs
- A graph is a finite set of *nodes* that are connected by *arcs*
- A *loop* may exist in a graph, where an arc lead back to the original node
- In general, such a graph is not explicitly given
- Search space is constructed during search

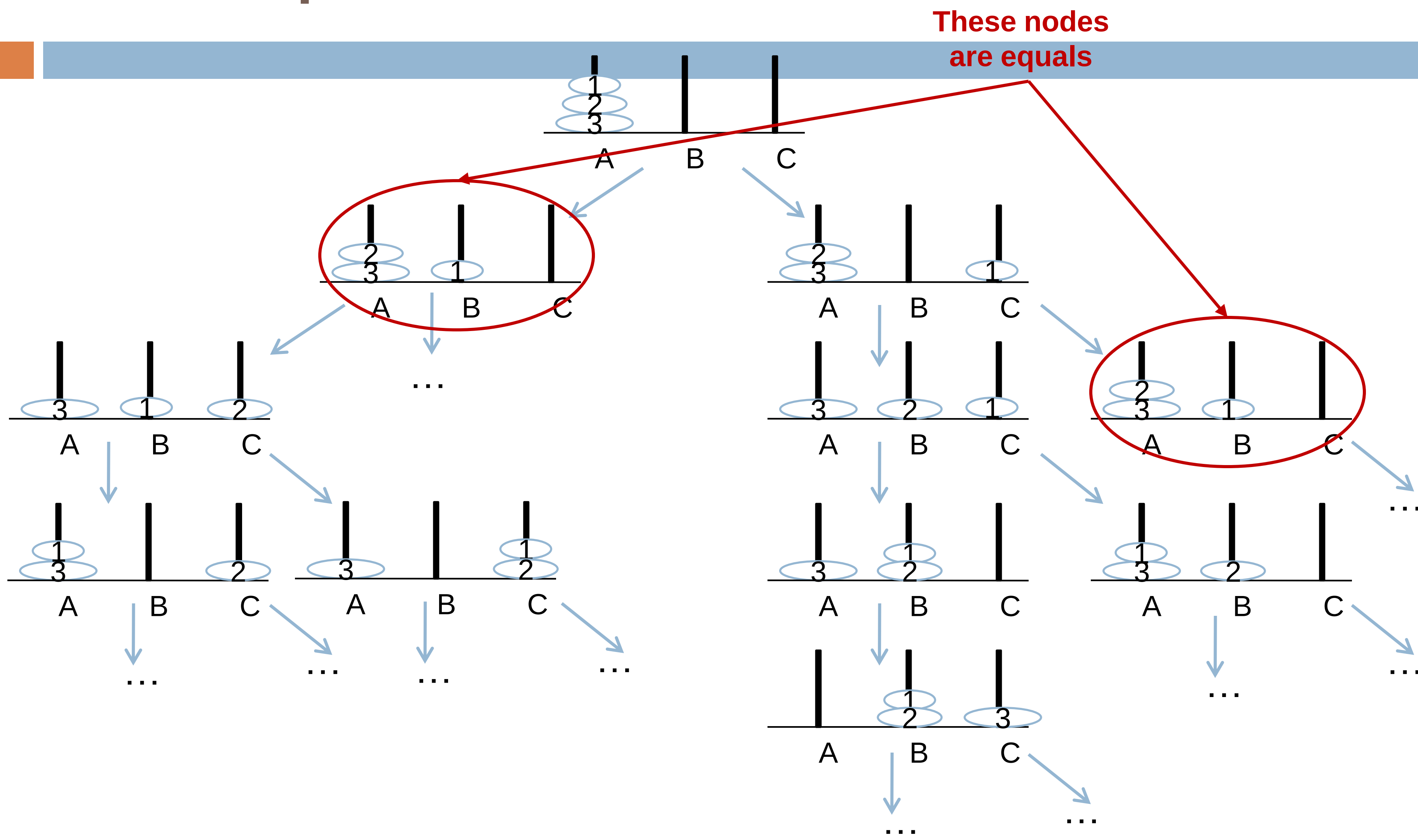


Search Space Representation

- A graph is *undirected* if arcs do not imply a direction, *direct* otherwise
- A graph is *connected* if every pair of nodes is connected by a path
- A connected graph with no loop is called *tree*
- A *weighted graph*, is a graph for which a value is associated to each arc

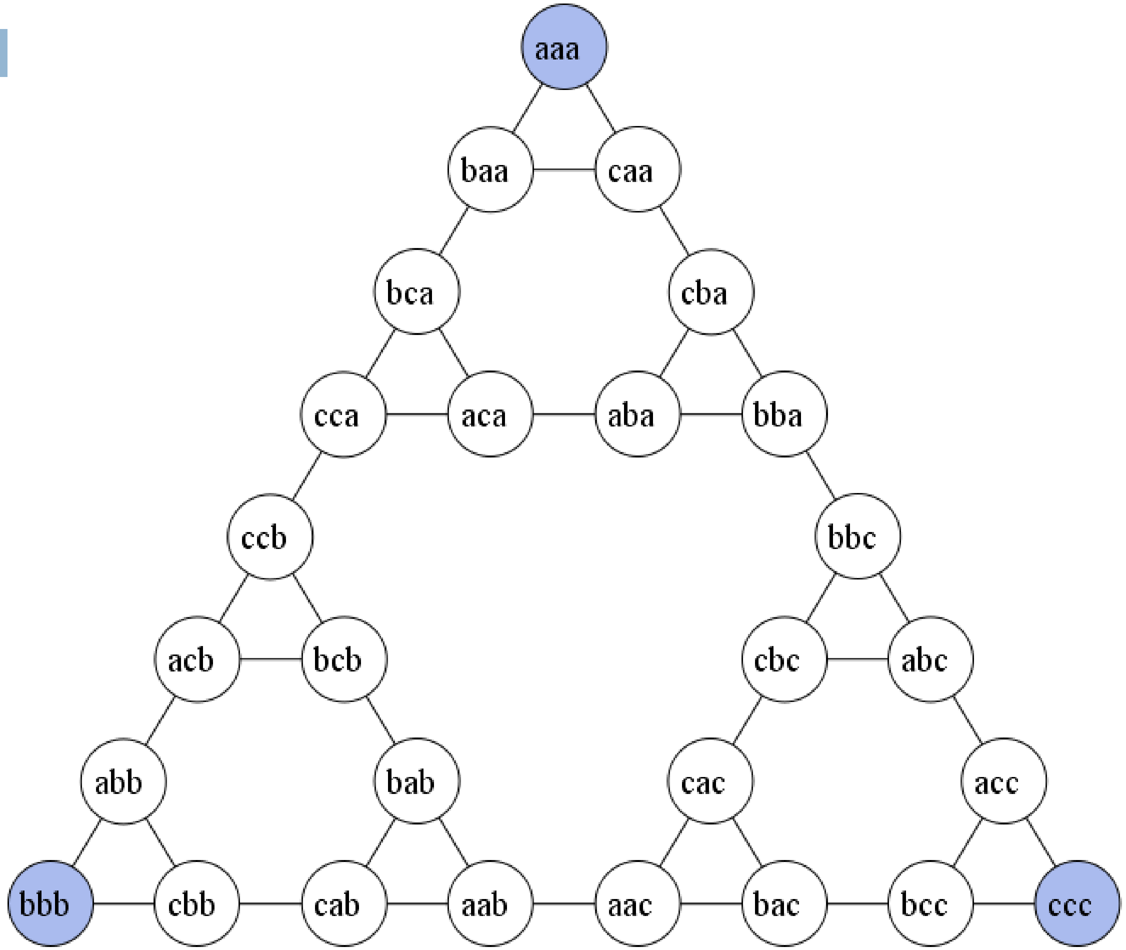


Example: Towers of Hanoi*



* A partial tree search space representation

Example: Towers of Hanoi*



* A complete direct graph representation
[http://en.wikipedia.org/wiki/Tower_of_Hanoi]

Search Methods

- A search method is defined by picking the order of node expansion

- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find the shortest path solution?

- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the shortest path solution
  - m: maximum depth of the state space (may be ∞)

Search Methods

- Uninformed techniques
  - Systematically search complete graph, unguided
  - Also known as brute force, naïve, or blind

- Informed methods
  - Use problem specific information to guide search in promising directions

UNINFORMED SEARCH

Brute force approach to explore search space

Uninformed Search

- A class of general purpose algorithms that operates in a brute force way
  - The search space is explored without leveraging on any information on the problem
- Also called blind search, or naïve search
- Since the methods are generic they are intrinsically inefficient

- E.g. Random Search
  - This method selects randomly a new state from the current one
  - If the goal state is reached, the search terminates
  - Otherwise the methods randomly select an other operator to move to the next state

- Prominent methods:
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

Depth-First Search

- Depth-First Search (DFS) begins at the root node and exhaustively search each branch to it maximum depth till a solution is found
  - The successor node is selected going in depth using from right to left (w.r.t. graph representing the search space)
- If greatest depth is reach with not solution, we backtrack till we find an unexplored branch to follow

- DFS is not complete
  - If cycles are presented in the graph, DFS will follow these cycles indefinitively
  - If there are no cycles, the algorithm is complete
  - Cycles effects can be limited by imposing a maximal depth of search (still the algorithm is incomplete)
- DFS is not optimal
  - The first solution is found and not the shortest path to a solution

- The algorithm can be implemented with a Last In First Out (LIFO) stack or recursion

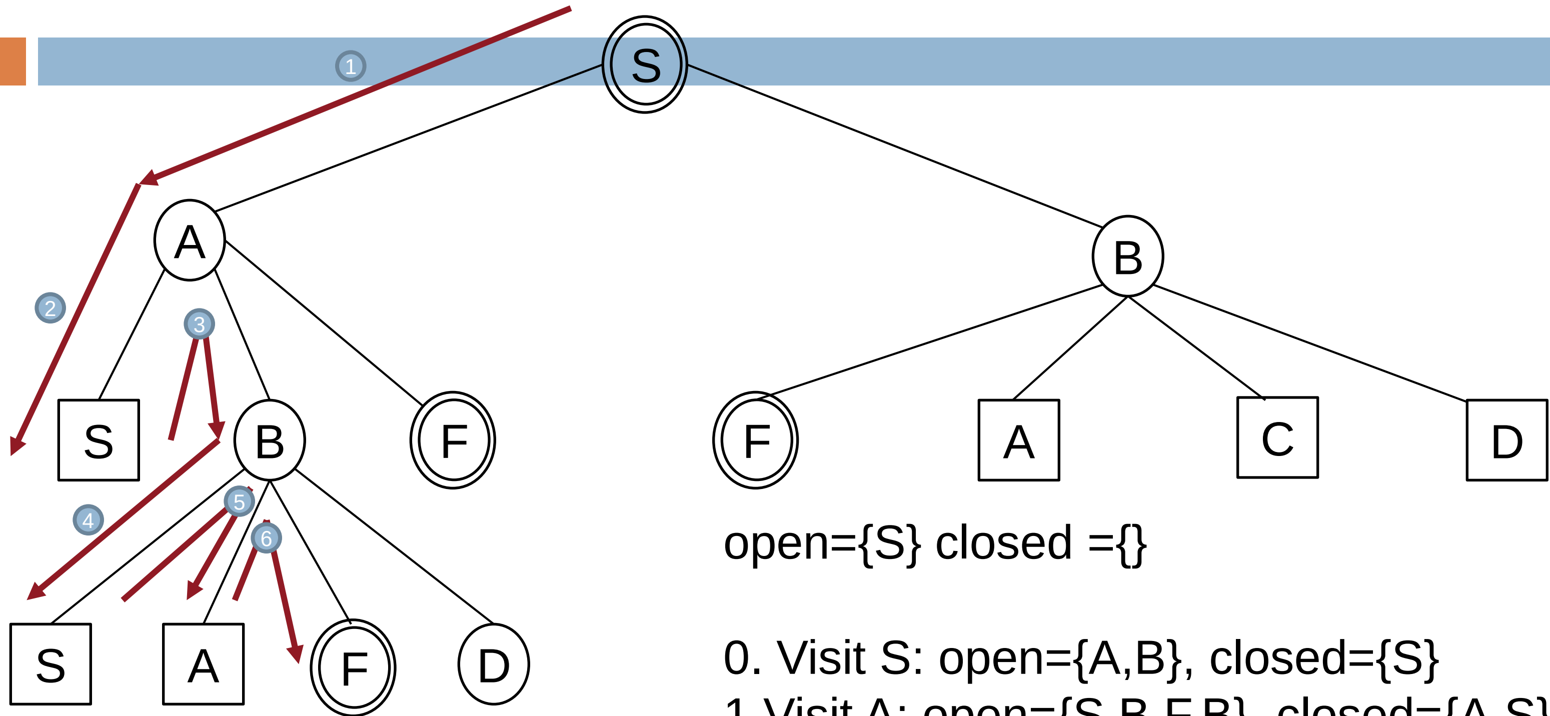
Depth-First Search: Algorithm

```
List open, closed, successors={};
Node root_node, current_node;
insert-first(root_node,open)

while not-empty(open);
    current_node= remove-first(open);
    insert-first (current_node,closed);
    if (goal(current_node)) return current_node;
    else
        successors=successorsOf(current_node);
        for(x in successors)
                if(not-in(x,closed)) insert-first(x,open);
    endIf
endWhile
```

N.B.= this version is not saving the path for simplicity

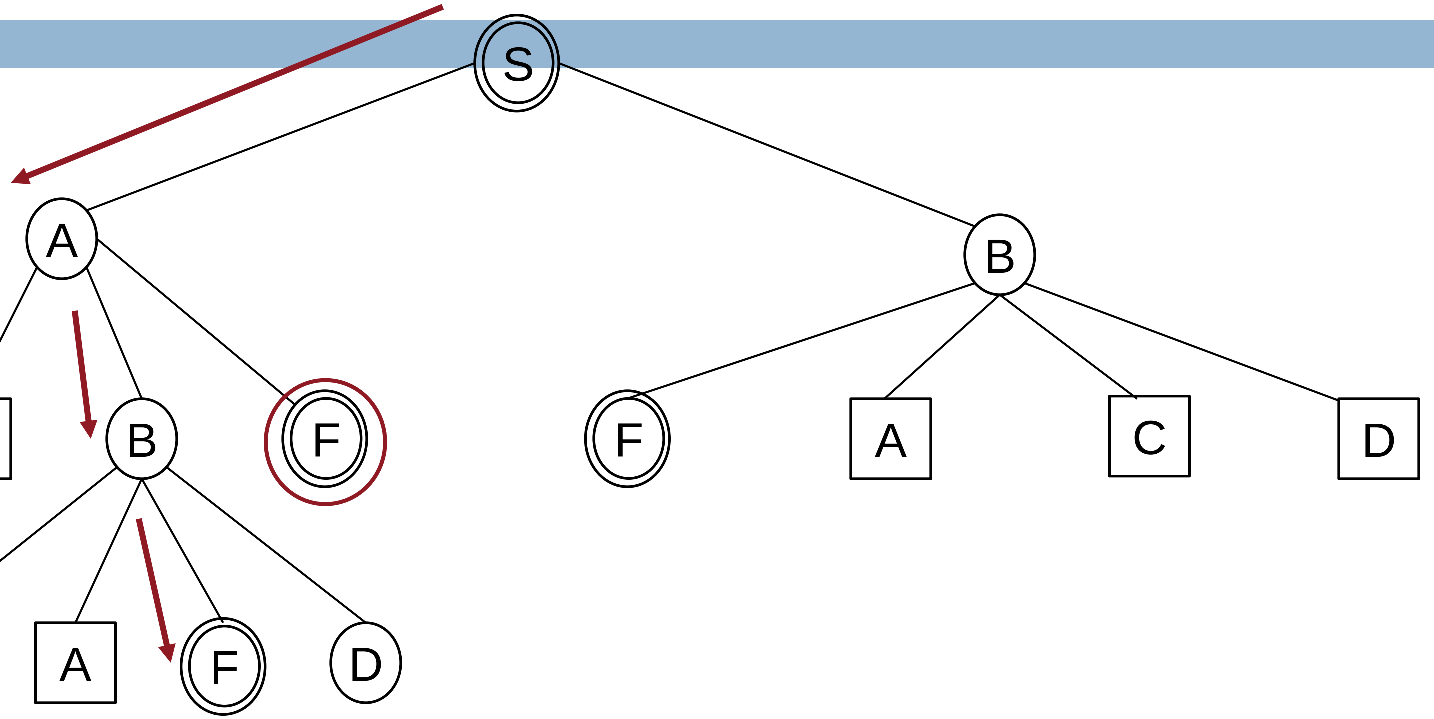
Depth-First Search: Example



open={S} closed ={}

0. Visit S: open={A,B}, closed={S}
1. Visit A: open={S,B,F,B}, closed={A,S}
2. Visit S: open={B,F,B}, closed={S,A,S}
3. Visit B: open={S,A,F,D,F,B}, closed={B,S,A,S}
4. Visit S: open={A,F,D,F,B}, closed={S,B,S,A,S}
5. Visit A: open={F,D,F,B}, closed={A,S,B,S,A,S}
6. Visit F: GOAL Reached!

Depth-First Search: Example



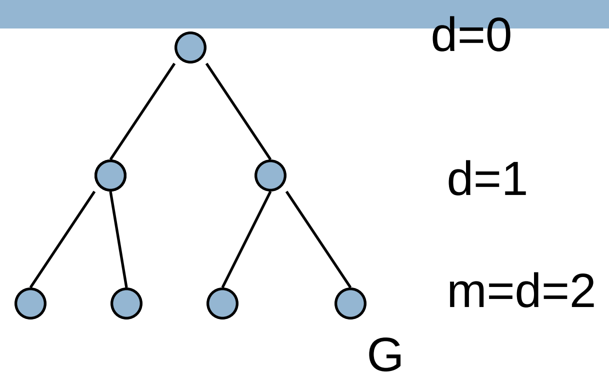
Result is: S->A->B->F

Depth-First Search: Complexity

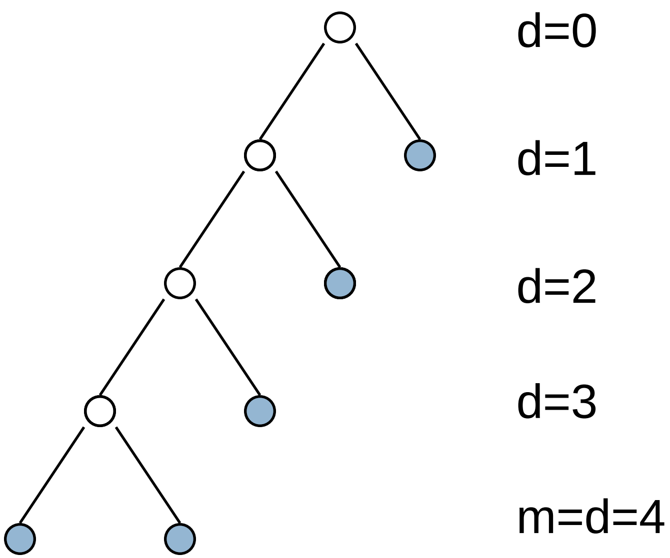
- Time Complexity
  - assume (worst case) that there is 1 goal leaf at the RHS
  - so DFS will expand all nodes

    $=1 + b + b^2 + \dots\dots\dots + b^m$

    $= \mathbf{O}\ (\mathbf{b^m})$
  - where **m** is the max depth of the tree



- Space Complexity
  - how many nodes can be in the queue (worst-case)?
  - at each depth l < d we have b-1 nodes
  - at depth m we have b nodes
  - total = (d-1)*(b-1) + b = **O(bm)**



Breadth-First Search

- Breadth-First Search (BFS) begins at the root node and explore level-wise al the branches

- BFS is complete
  - If there is a solution, BFS will found it
- BFS is optimal
  - The solution found is guaranteed to be the shortest path possible

- The algorithm can be implemented with a First In First Out (FIFO) stack

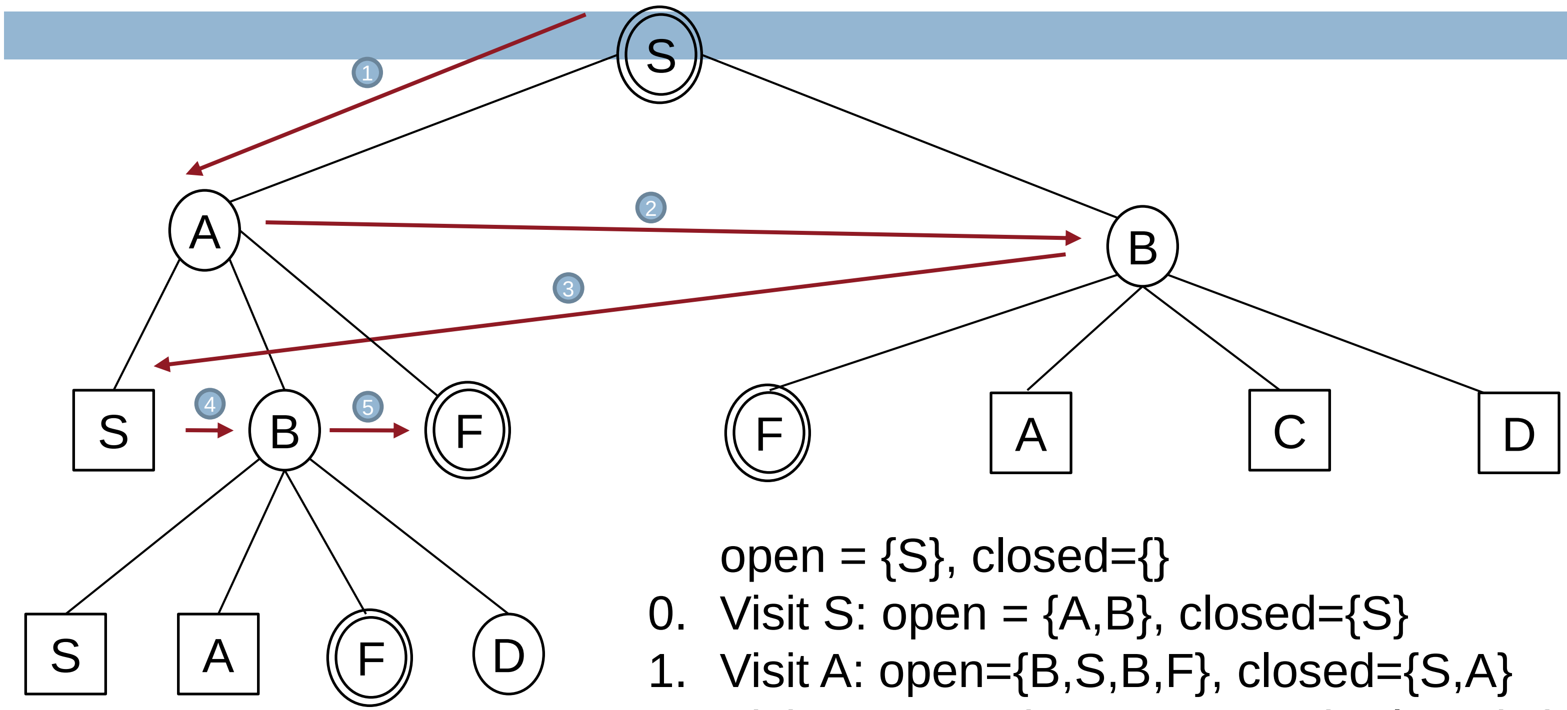
Breadth-First Search: Algorithm

```
List open, closed, successors={};
Node root_node, current_node;
insert-last(root_node,open)

while not-empty(open);
    current_node=remove-first(open);
    insert-last(current_node,closed);
    if (goal(current_node)) return current_node;
    else
        successors=successorsOf(current_node);
        for(x in successors)
                if(not-in(x,closed)) insert-last(x,open);
    endIf
endWhile
```

N.B.= this version is not saving the path for simplicity

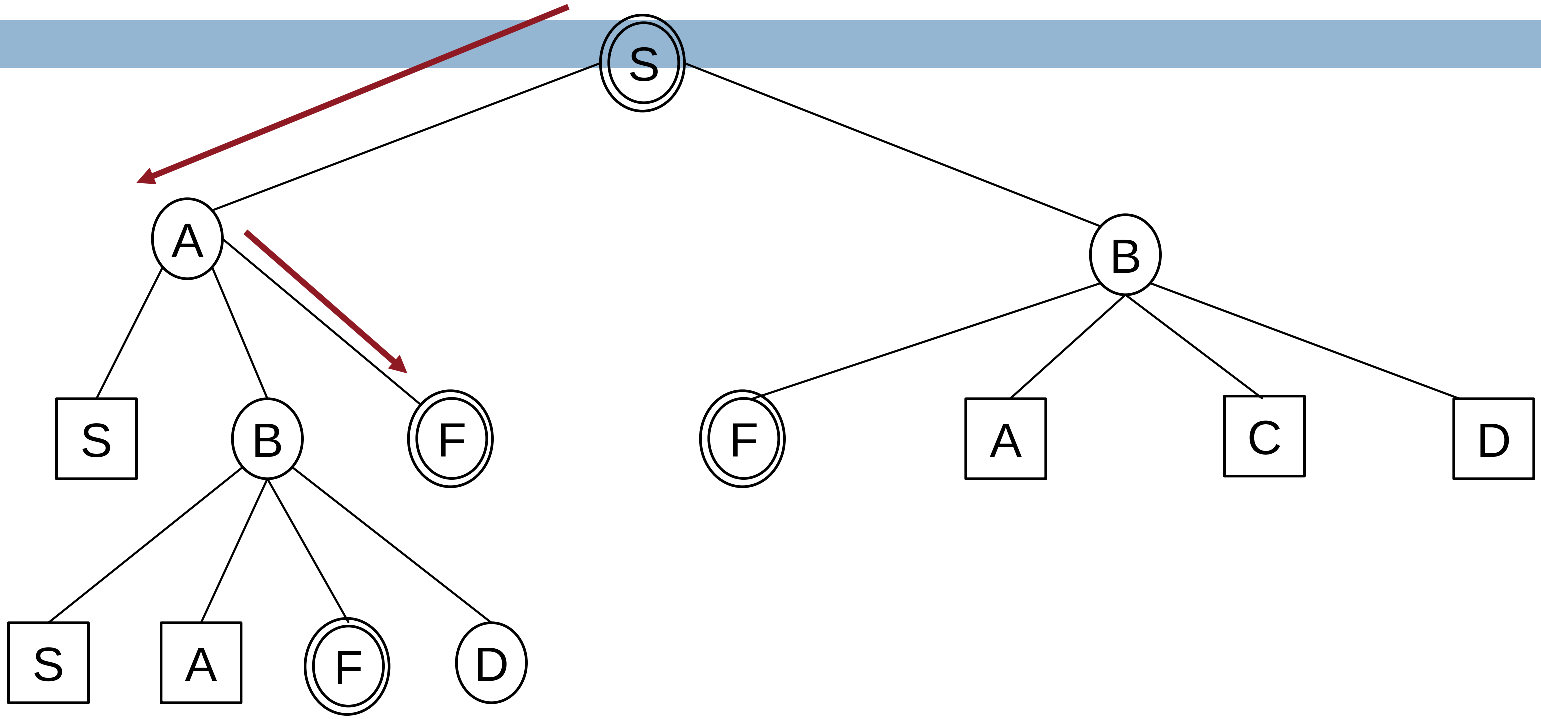
Breadth-First Search: Example



open = {S}, closed={}

0. Visit S: open = {A,B}, closed={S}
1. Visit A: open={B,S,B,F}, closed={S,A}
2. Visit B: open={S,B,F,F,A,C,D}, closed={S,A,B}
3. Visit S: open={B,F,F,A,C,D}, closed={S,A,B,S}
4. Visit B: open={F,F,A,C,D,S,A,C,D}, closed={S,A,B,S,B}
5. Visit F: Goal Found!

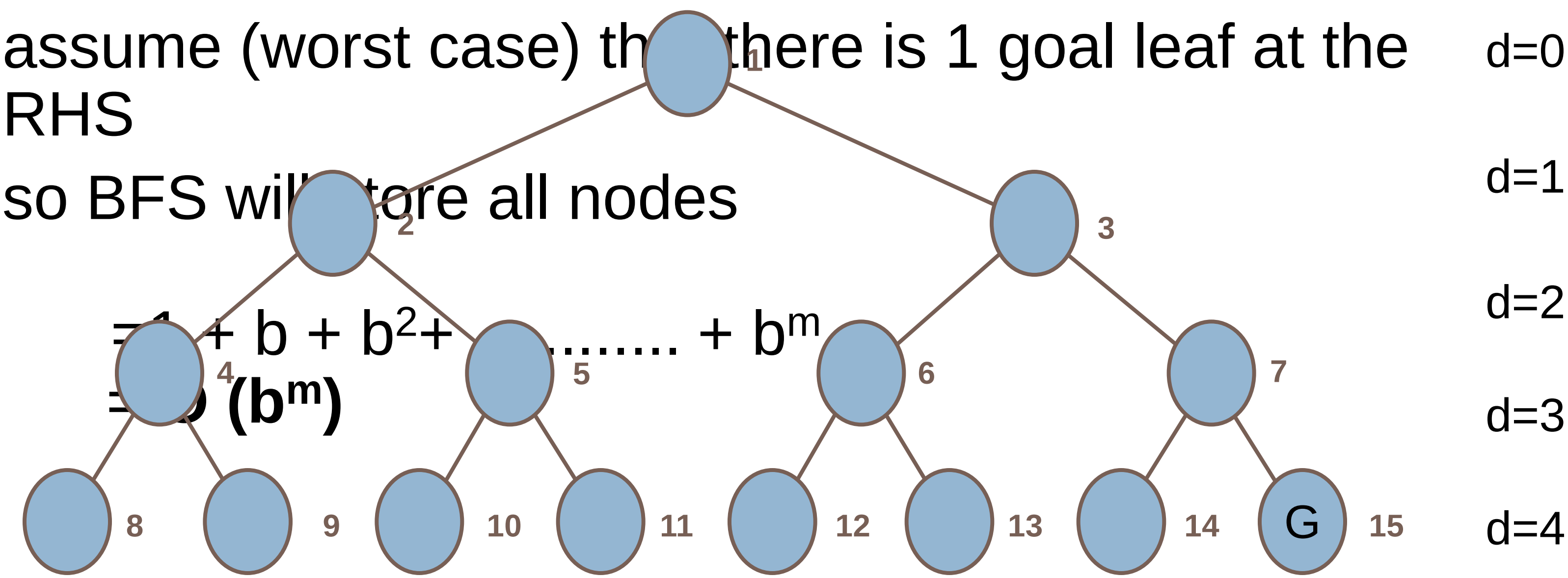
Breadth-First Search: Example



Result is: S->A->F

Breadth-First Search: Complexity

- Time complexity is the same magnitude as DFS
  - **O (b^m)**
  - where **m** is the depth of the solution
- Space Complexity
  - how many nodes can be in the queue (worst-case)?
  - assume (worst case) that there is 1 goal leaf at the RHS
  - so BFS will store all nodes



$= 1 + b + b^2 + \ldots\ldots + b^m$

$= \mathbf{O\ (b^m)}$

Further Uninformed Search Strategies

- *Depth-limited search (DLS)*: Impose a cut-off (e.g. n for searching a path of length n-1), expand nodes with max. depth first until cut-off depth is reached (LIFO strategy, since it is a variation of depth-first search).
- *Bidirectional search (BIDI)*: forward search from initial state & backward search from goal state, stop when the two searches meet. Average effort $O(b^{d/2})$ if testing whether the search fronts intersect has constant effort
- In AI, the problem graph is typically not known. If the graph is known, to find *all* optimal paths in a graph with labelled arcs, *standard graph algorithms* can be used

INFORMED SEARCH

Using knowledge on the search space to reduce search costs

Informed Search

- Blind search methods take $O(b^m)$ in the worst case
- May make blind search algorithms prohibitively slow where d is large
- How can we reduce the running time?
  - Use problem-specific knowledge to pick which states are better candidates

Informed Search

- Also called heuristic search
- In a heuristic search each state is assigned a “heuristic value” (h-value) that the search uses in selecting the “best” next step
- A heuristic is an operationally-effective nugget of information on how to direct search in a problem space
- Heuristics are only approximately correct

Informed Search: Prominent methods

- Best-First Search
- A*
- Hill Climbing

Cost and Cost Estimation

$$f(n)=g(n)+h(n)$$

- g(n) the cost (so far) to reach the node n
- h(n) estimated cost to get from the node to the goal
- f(n) estimated total cost of path through n to goal

Informed Search: Best-First Search

- Special case of breadth-first search
- Uses h(n) = heuristic function as its evaluation function
- Ignores cost so far to get to that node (g(n))
- Expand the node that *appears* closest to goal

- Best First Search is complete
- Best First Search is not optimal
  - A solution can be found in a longer path (higher h(n) with a lower g(n) value)

- Special cases:
  - uniform cost search: f(n) = g(n) = path to n
  - A^* search

Best-First Search: Algorithm

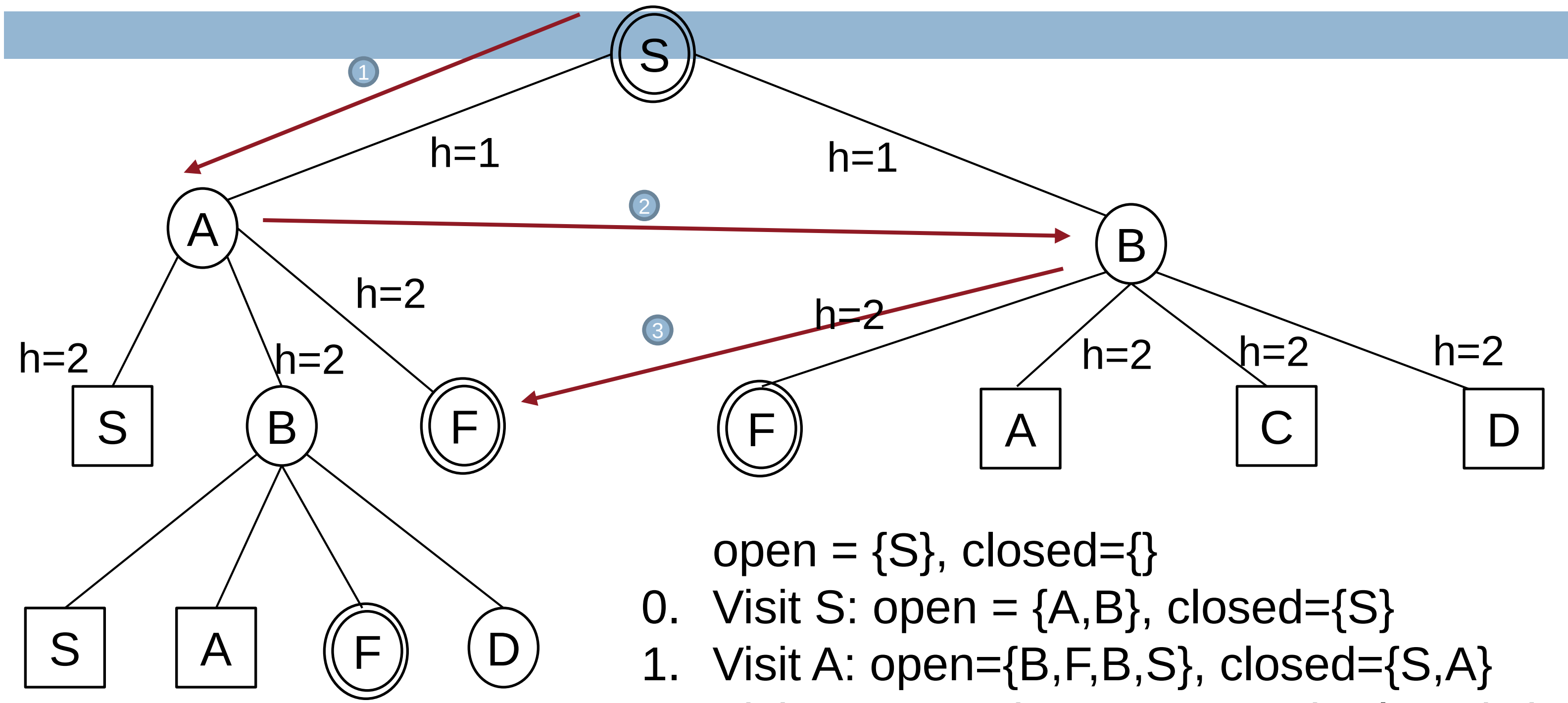
```
List open, closed, successors={};
Node root_node, current_node;
insert-last(root_node,open)

while not-empty(open);
      current_node=remove-first (open);
      insert-last(current_node,closed);
      if (goal(current_node)) return current_node;
      else
            successors=estimationOrderedSuccessor (current_node);
            for(x in successors)
                        if(not-in(x,closed)) insert-last(x,open);
      endIf
endWhile
```

returns the list of direct descendants of the current node in shortest cost order

N.B.= this version is not saving the path for simplicity

Best-First Search: Example

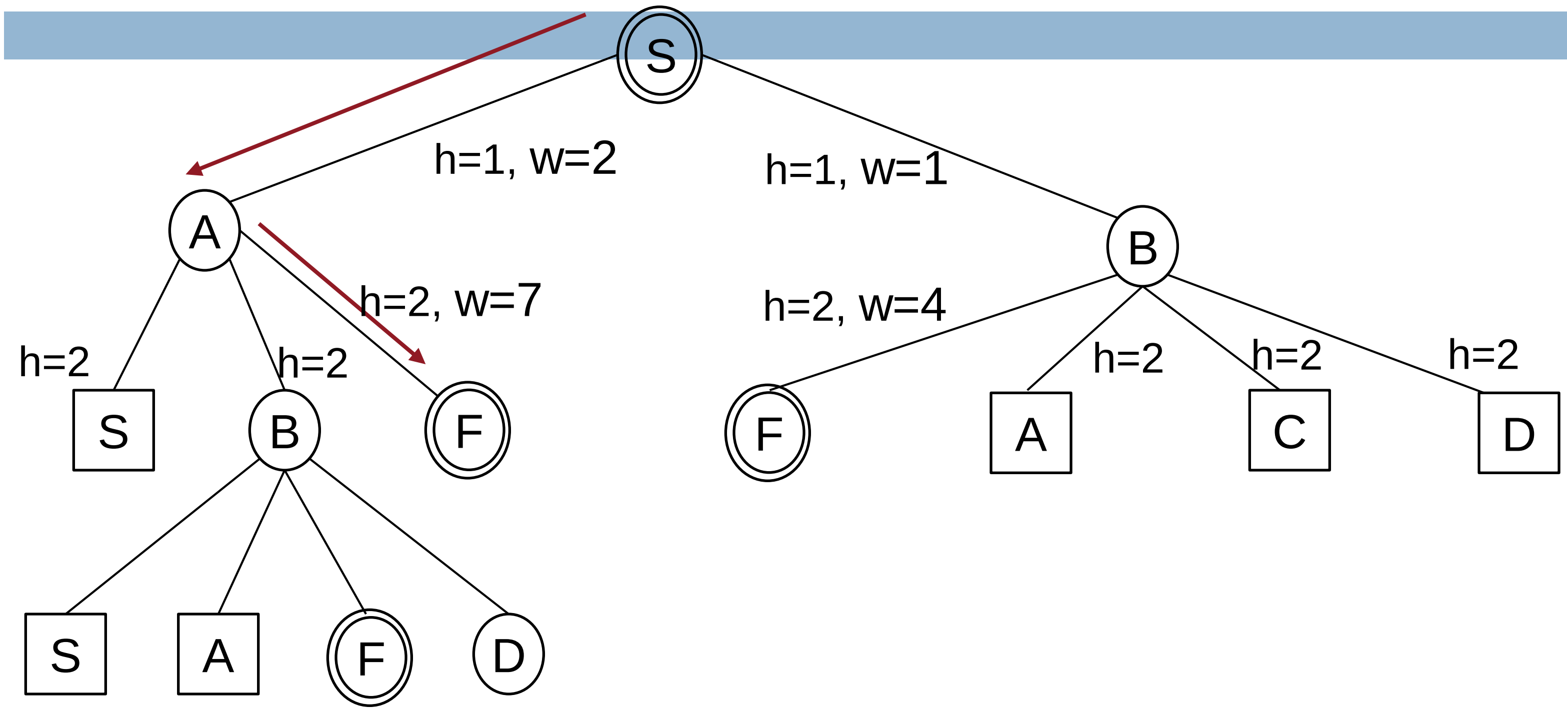


open = {S}, closed={}

0. Visit S: open = {A,B}, closed={S}
1. Visit A: open={B,F,B,S}, closed={S,A}
2. Visit B: open={F,B,S,F,A,C,D}, closed={S,A,B}
3. Visit F: Goal Found!

In this case we estimate the cost as the distance from the root node (in term of nodes)

Best-First Search: Example



Result is: S->A->F!

If we consider real costs, optimal solution is:
S->B->F

A*

- Derived from Best-First Search
- Uses both g(n) and h(n)
- A* is optimal
- A* is complete

A* : Algorithm

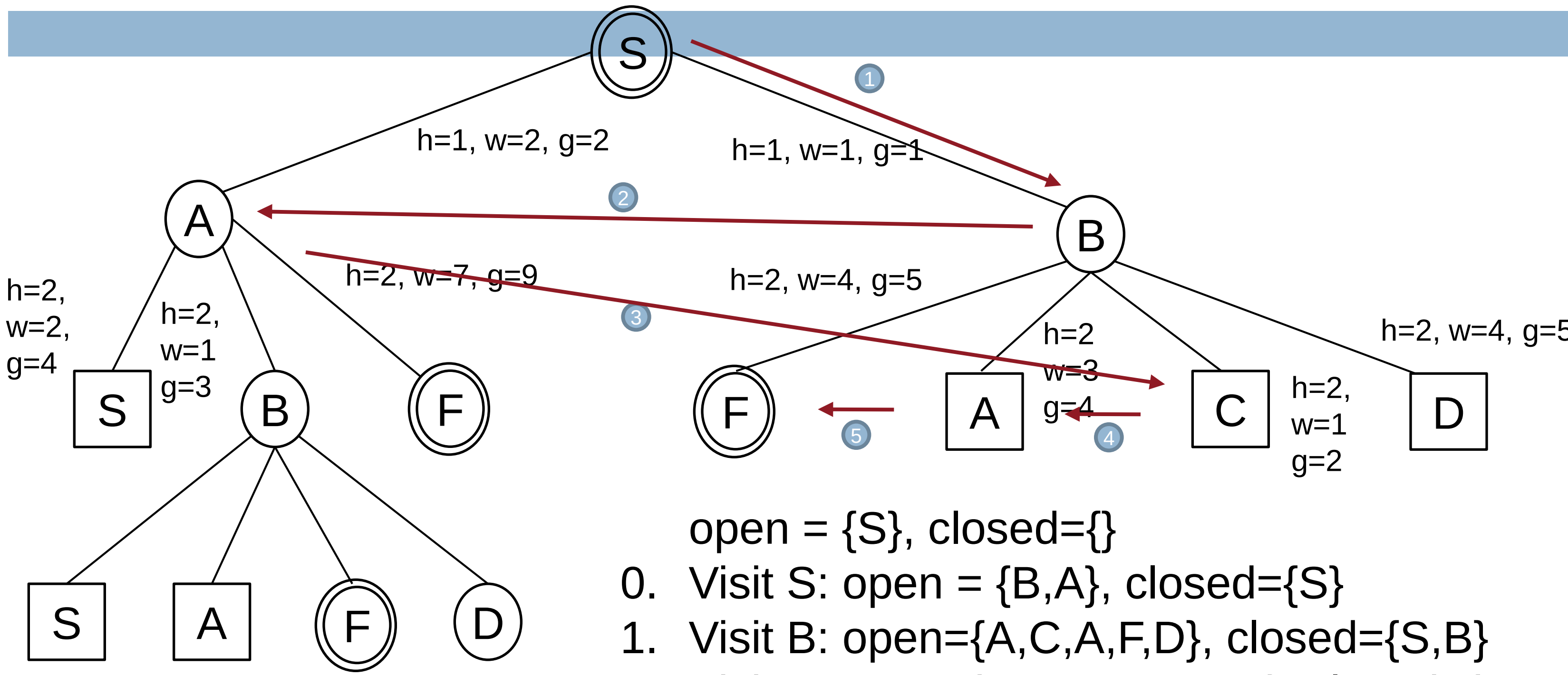
```
List open, closed, successors={};
Node root_node, current_node, goal;
insert-back(root_node,open)

while not-empty(open);
    current_node=remove-front(open);
    insert-back(current_node,closed);
    if (current_node==goal) return current_node;
    else
        successors=totalEstOrderedSuccessors(current_node);
        for(x in successors)
                if(not-in(x,closed)) insert-back(x,open);
    endIf
endWhile
```

returns the list of direct descendants of the current node in shortest total estimation order

N.B.= this version is not saving the path for simplicity

A* : Example

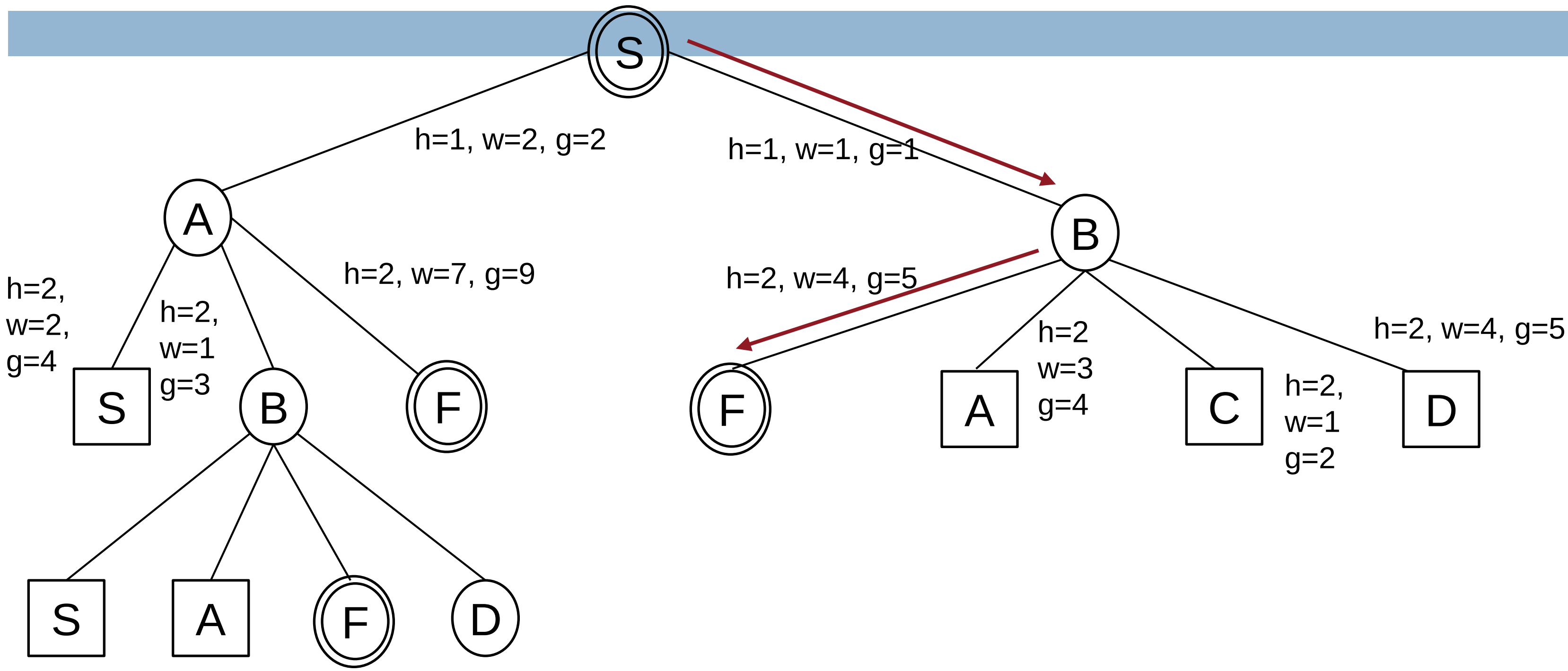


open = {S}, closed={}

0. Visit S: open = {B,A}, closed={S}
1. Visit B: open={A,C,A,F,D}, closed={S,B}
2. Visit A: open={C,A,F,D,B,S,F}, closed={S,B,A}
3. Visit C: open={A,F,D,B,S,F}, closed={S,B,A,C}
4. Visit A: open={F,D,B,S,F}, closed={S,B,A,C,A}
5. Visit F: Goal Found!

In this case we estimate the cost as the distance from the root node (in term of nodes)

A* : Example



Result is: S->B->F!

Hill Climbing

- Special case of depth-first search
- Uses h(n) = heuristic function as its evaluation function
- Ignores cost so far to get to that node (g(n))
- Expand the node that *appears* closest to goal

- Hill Climbing is not complete
  - Unless we introduce backtracking
- Hill Climbing is not optimal
  - Solution found is a local optimum

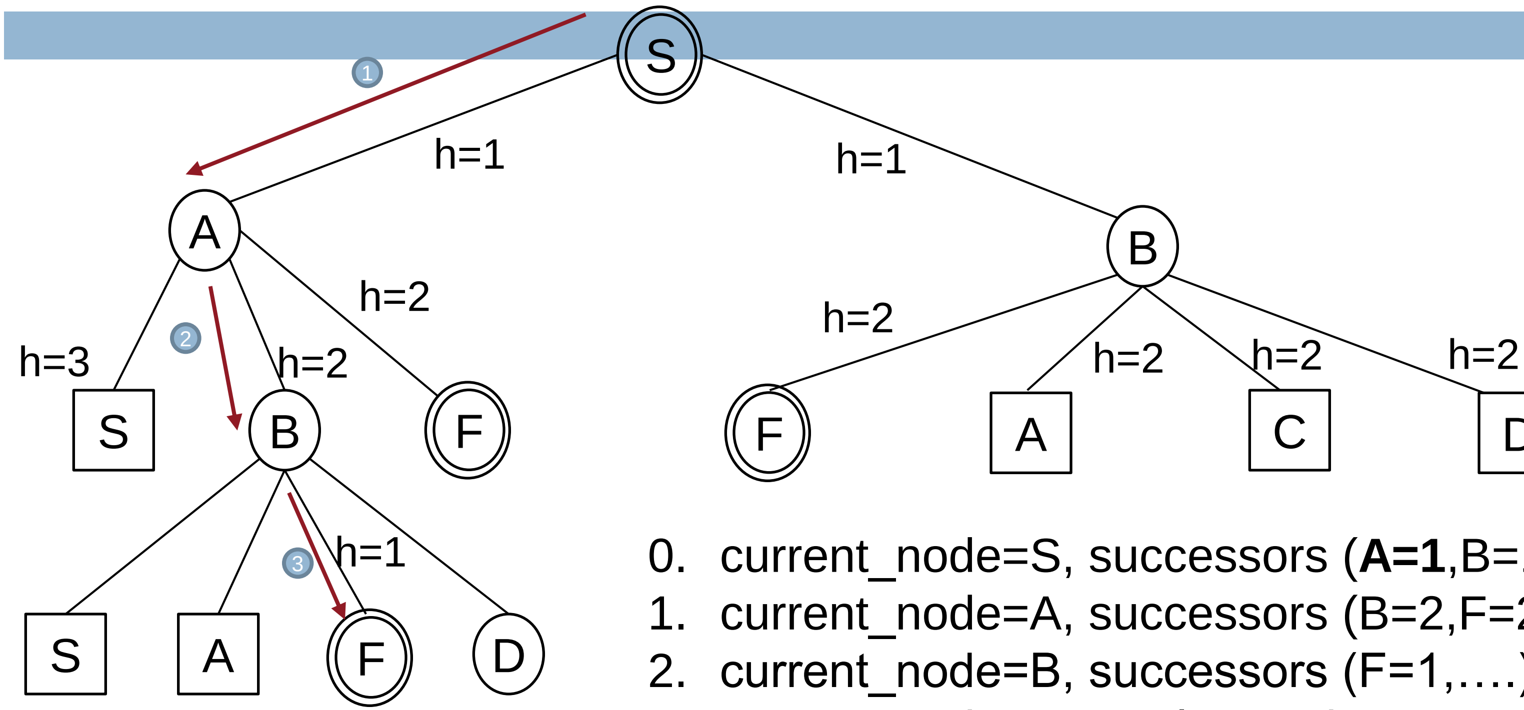
Hill Climbing: Algorithm

```
List successors={}; Node root_node, current_node, nextNode;

current_node=root_node
while (current_node!=null)
    if (goal(current_node)) return current_node;
    else
        successors=successorsOf(current_node);
        nextEval = -∞; nextNode=null;
        for(x in successors)
                if(eval(x)> nextEval)
                        nexEval=eval(x);
                        nextNode=x;
        current_node=nextNode,
    endIf
endWhile
```

N.B.= this version is not using backtracking

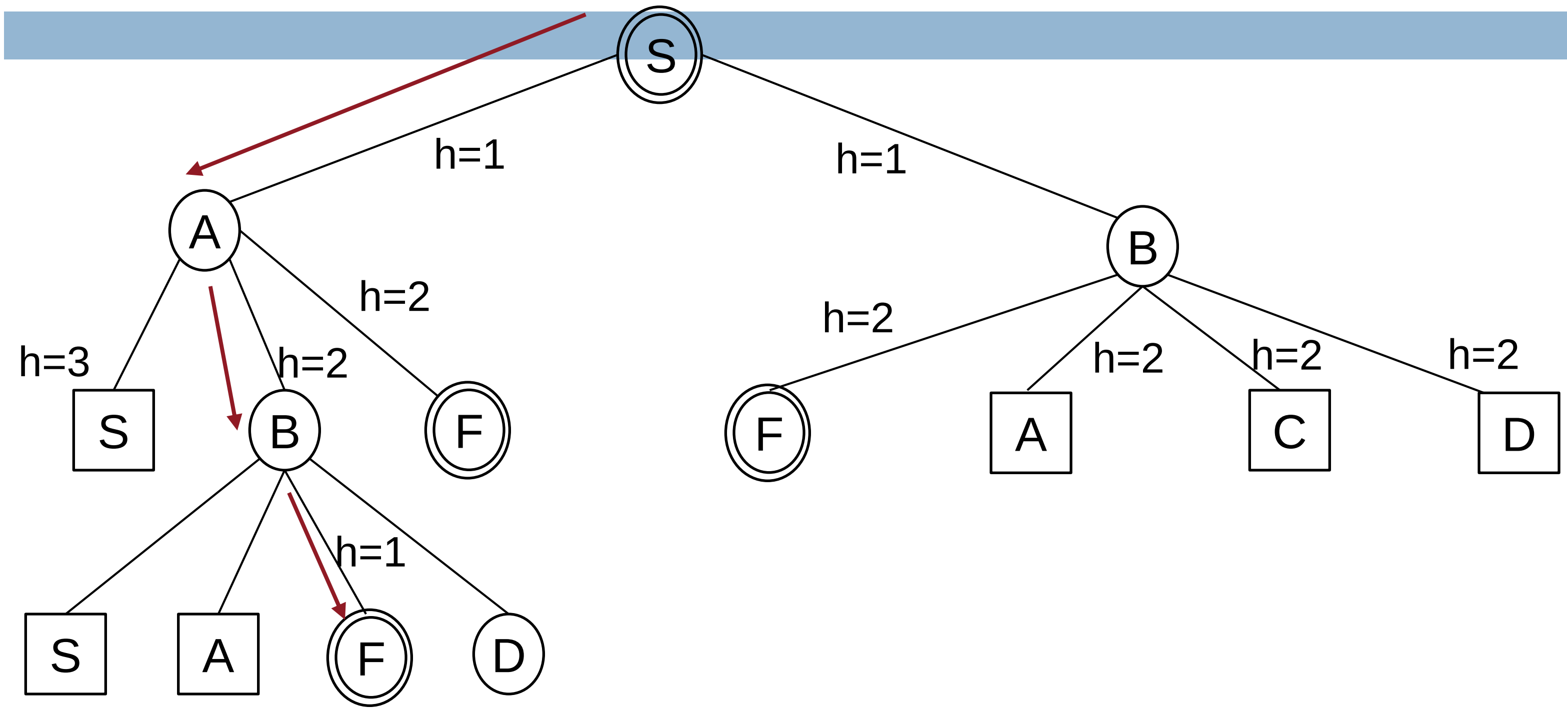
Hill Climbing: Example



0. current_node=S, successors (**A=1**,B=1)
1. current_node=A, successors (B=2,F=2,S=2)
2. current_node=B, successors (F=1,….)
3. current_node=F: Goal Found!

In this case we estimate the cost as the distance from the root node (in term of nodes)

Hill Climbing: Example



Result is: S->A->B->F!

Not optimal, more if at step 1 h(S)=2 we would have completed without funding a solution

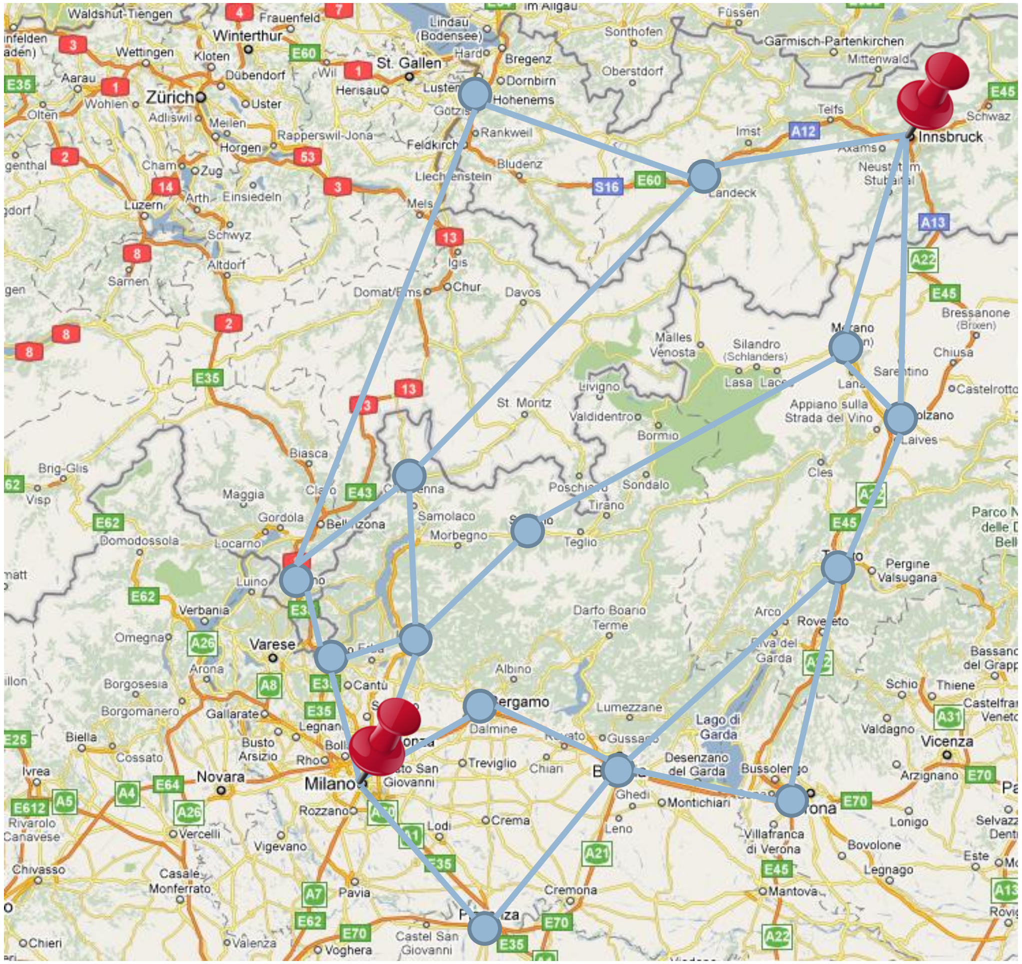
Informed Search Algorithm Comparison

Algorithm	Time	Space	Optimal	Complete	Derivative
Best First Search	O(bm)	O(bm)	No	Yes	BFS
Hill Climbing	$O(\infty)$	O(b)	No	No	
A*	$O(2^N)$	$O(b^d)$	Yes	Yes	Best First Search

b, branching factor
d, tree depth of the solution
m, maximum tree depth

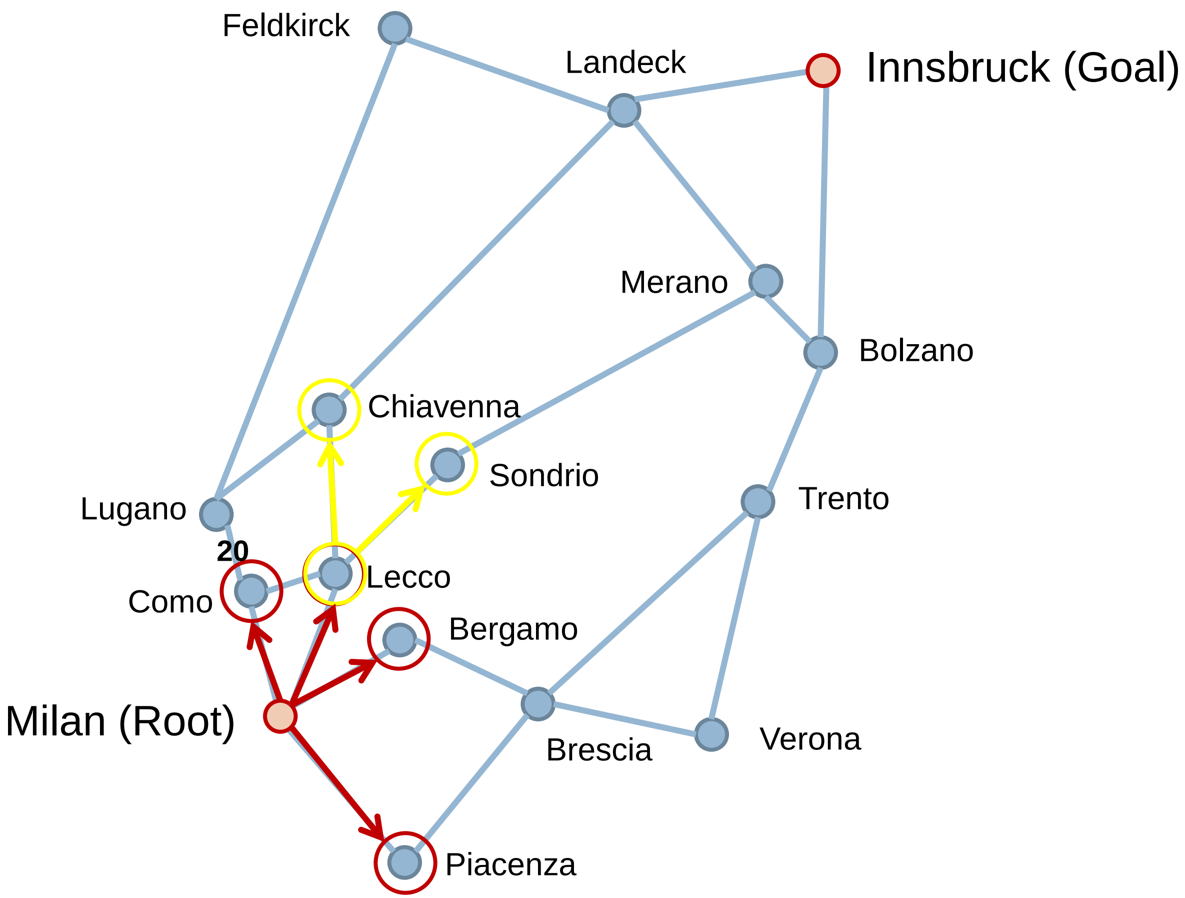
ILLUSTRATION BY A LARGER EXAMPLE

Route Search



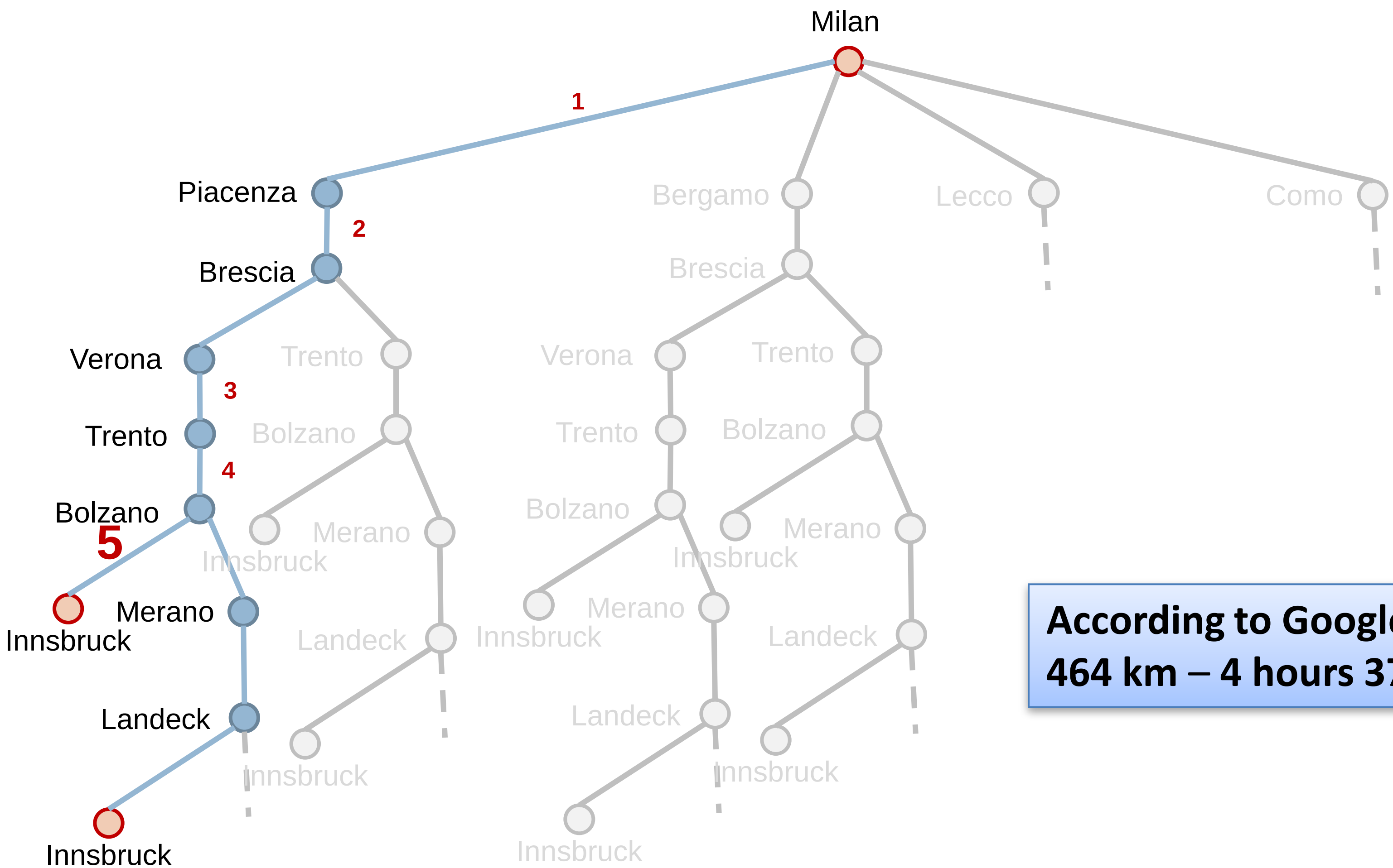
- Start point: Milan
- End point: Innsbruck
- Search space: Cities
  - Nodes: Cities
  - Arcs: Roads
- Let's find a possible route!

Graph Representation



- We start from the root node, and pick the leaves
- The same apply to each leaves
  - But we do not reconsider already used arcs
- The first node picked is the first node on the right

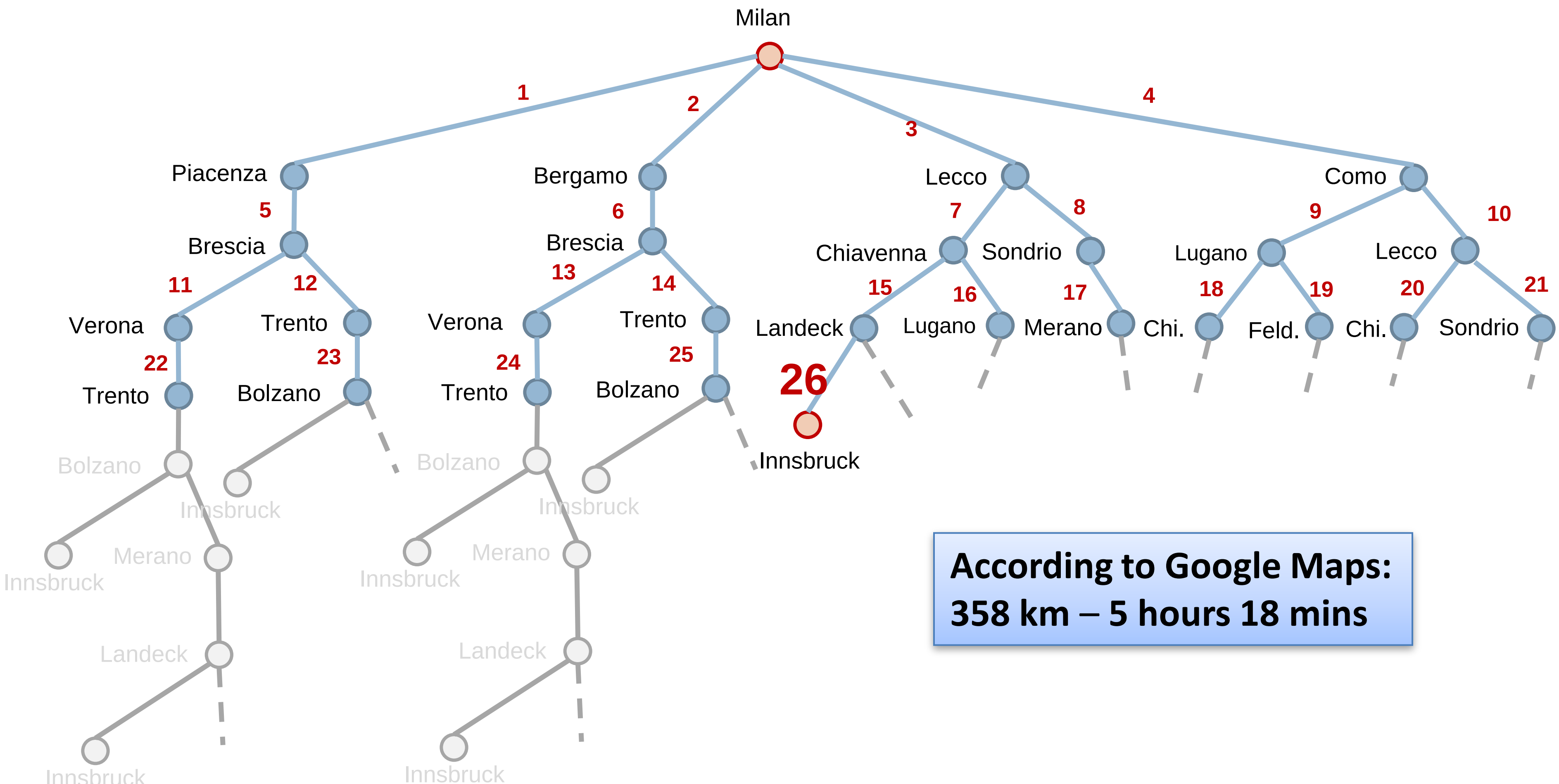
Depth-First Search



According to Google Maps:
464 km – 4 hours 37 mins

N.B.: by building the tree, we are exploring the search space!

Breadth-First search



According to Google Maps:
358 km – 5 hours 18 mins

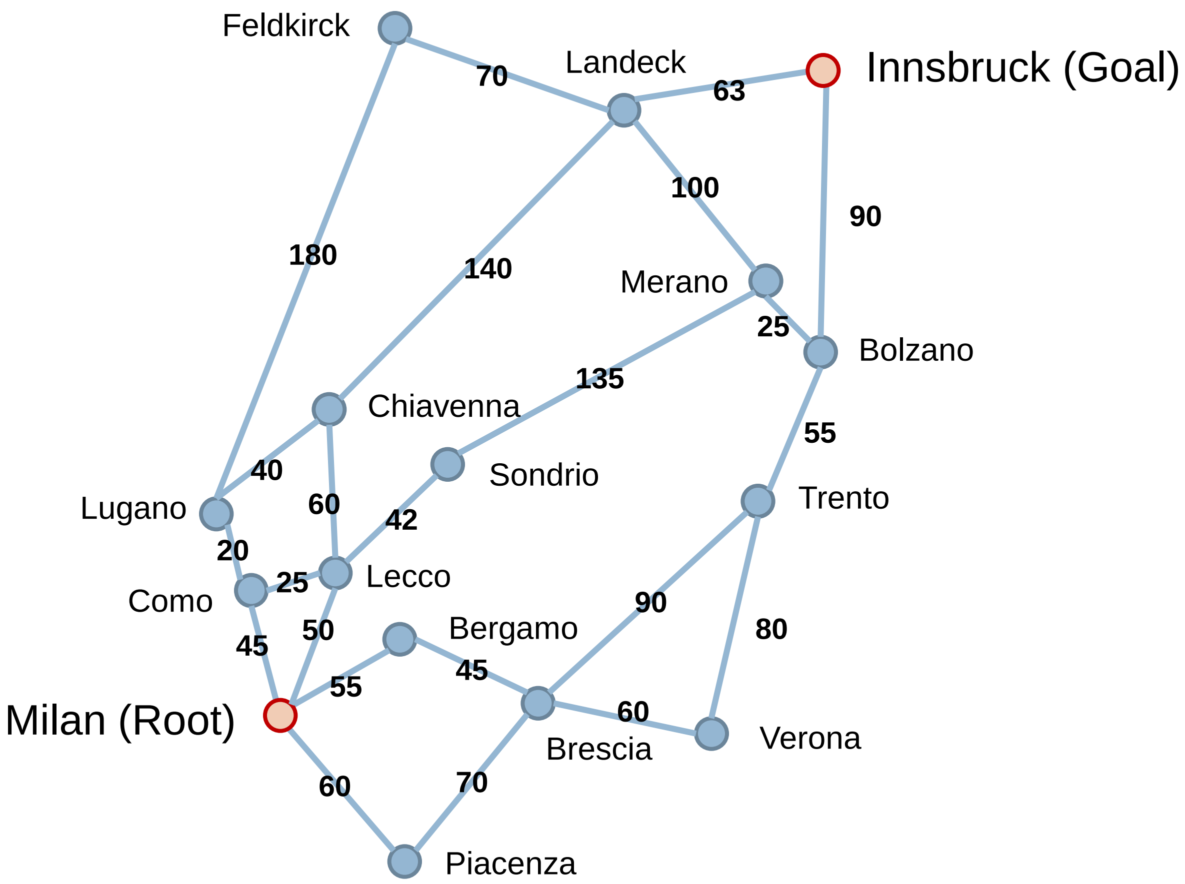
N.B.: by building the tree, we are exploring the search space!

Depth-First Search vs Breadth-First search

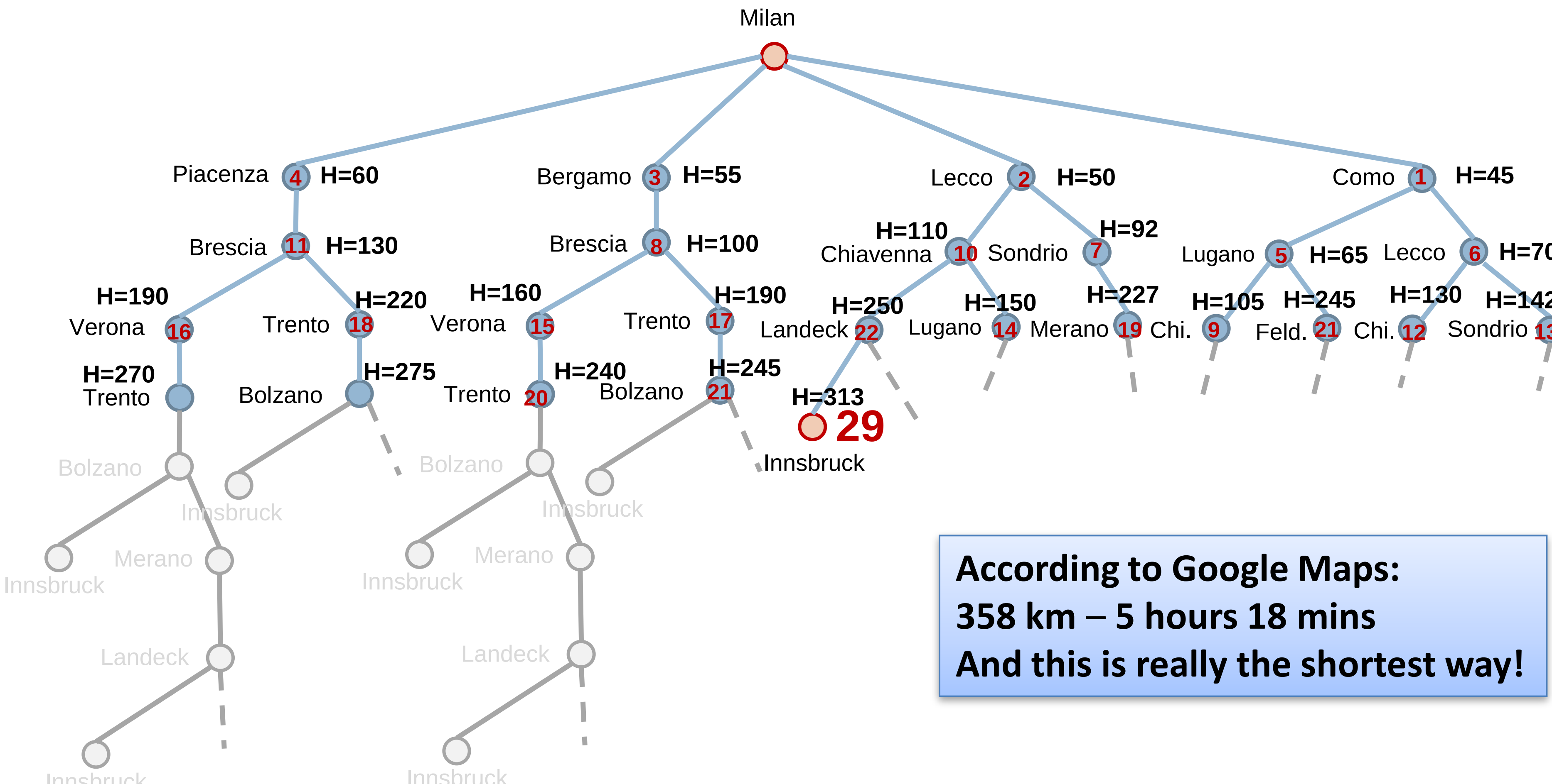
- Distance
  - DFS: 464 km
  - BFS: 358 km
  - Q1: Can we use an algorithm to optimize according to distance?
- Time
  - DFS: 4 hours 37 mins
  - BFS: 5 hours 18 mins
  - Q2: Can we use an algorithm to optimize according to time?
- Search space:
  - DFS: 5 expansions
  - BFS: 26 expansions
  - Not very relevant… depends a lot on how you pick the order of node expansion, never the less BFS is usually more expensive

- To solve Q1 and Q2 we can apply for example and Best-First Search
  - Q1: the heuristic maybe the air distance between cities
  - Q2: the heuristic maybe the air distance between cities x average speed (e.g. 90km/h)

Graph Representation with approximate distance



Best-First search



According to Google Maps:
358 km – 5 hours 18 mins
And this is really the shortest way!

N.B.: by building the tree, we are exploring the search space!

EXTENSIONS

Variants to presented algorithms

- Combine Depth First Search and Breadth First Search, by performing Depth Limited Search with increased depths until a goal is found
- Enrich Hill Climbing with random restart to hinder the local maximum and foothill problems
- Stochastic Beam Search: select w nodes randomly; nodes with higher values have a higher probability of selection
- Genetic Algorithms: generate nodes like in stochastic beam search, but from two parents rather than from one

SUMMARY

Summary

- Uninformed Search
  - If the branching factor is small, BFS is the best solution
  - If the tree is depth IDS is a good choice

- Informed Search
  - Heuristic function selection determines the efficiency of the algorithm
  - If actual cost is very expensive to be computed, then Best First Search is a good solution
  - Hill climbing tends to stack in local optimal solutions